

Practice Test D: Answer Key and Rubric

Tutor copy. Every answer was checked by symbolic computation (SymPy). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Part I, Part II, Q13, Q16): all or nothing.
- **Infinite limits.** The instruction box tells students to write ∞ or $-\infty$. “DNE” for a limit whose answer is ∞ or $-\infty$ earns 1 point of the item’s value. A wrong sign earns 0. Where one side is ∞ and the other $-\infty$, the answer is DNE and “ ∞ ” earns 0.
- **Function-value items** (Q1 value parts, Q2(c), Q7(e), Q8, Q13): writing the *limit* where the function value was asked earns 0. “Undefined,” “not defined,” or “does not exist” are all accepted for an undefined value.
- **Show-work items** (Parts III–V): a bare correct answer earns at most half credit.
- **Method is not graded for efficiency.** A correct answer reached by long but valid algebra gets full credit. The time it costs is the penalty.

Test D

Item	Answer	Rubric / trap
1(a)	$-\infty$	
1(b)	$-\infty$	Both sides $-\infty$. <i>Trap</i> : reflexive “DNE” at an asymptote (1 pt).
1(c)	undefined	
1(d)	2	Open dot at (1, 2).
1(e)	DNE	Left 4, right 2.
1(f)	-1	<i>Trap</i> : function value; closed dot at $(-2, -1)$, not the hole at $(-2, 1)$.
1(g)	1	
2(a)	$-\infty$	Values $-10, -100, -1000$ decrease without bound.
2(b)	DNE	Left $-\infty$, right ∞ .
2(c)	0	Read directly from the table.
3	$\frac{5}{6}$	Direct substitution: $\frac{4-9}{4+2-12} = \frac{-5}{-6}$. <i>Trap</i> : the drilled version approaches 3; this one approaches 2. Factoring gives $\frac{t+3}{t+4} \rightarrow \frac{5}{6}$ (full credit, slower).
4	0	Direct: $\frac{2-2}{2} = 0$. <i>Trap</i> : the drilled version has denominator $x - 1$. “0/0”, “undefined”, “DNE”: 0 pts.
5	12	1 pt notes 0/0; 3 pts synthetic division by -2 on 1, 0, 0, 8 giving $x^2 - 2x + 4$, and denominator $(x + 2)(x + 3)$; 2 pts $\frac{4+4+4}{1}$. <i>Traps</i> : two zero placeholders; dividing by 2 instead of -2 . Sum-of-cubes factoring is also full credit.
6	(a) ∞ (b) ∞ (c) ∞	3 each. Numerator $\rightarrow 5$, $(x - 4)^2 > 0$ on both sides. <i>Trap</i> : “DNE” for (c) (1 pt).
7	(a) -8 (b) -2 (c) 2 (d) DNE (e) -2	2 each. <i>Trap in (a)</i> : $t = -3 \leq -1$ uses $3t + 1$; $t^2 + 1$ gives 10. <i>Trap in (e)</i> : $t \leq -1$ uses $3t + 1$.
8	undefined	$v(-1) = \frac{0}{0}$. <i>Trap</i> : dividing gives $x^2 - x + 1 \rightarrow 3$, the limit, not $v(-1)$. Answer 3 earns 0.

Item	Answer	Rubric / trap
9	Continuous	(1) $f(0) = 3$; (2) $\lim_{x \rightarrow 0} f(x) = 3$; (3) equal. 2 pts per condition; all three must be checked even though the answer is Yes.
10	Not continuous	(1) $k(3) = 9 - 3 - 3 = 3$; (2) left 3, right $6 - 4 = 2$, so the limit does not exist; (3) fails as a result. 2 pts per condition.
11	Yes	On $[-1, 1]$, $f(x) = x + 3$ with the closed dot at $(1, 4)$. $\lim_{x \rightarrow 1^-} f(x) = 4 = f(1)$, so f is continuous from the left at the endpoint. The jump at 1 is on the right, outside the interval; the hole at -2 is also outside. 2 answer, 2 reason.
12	(a) No (b) Yes	Denominator $(2x - 1)(x - 3)$ is zero at $\frac{1}{2}$ and 3. (a) $\frac{1}{2} \in [0, 1]$. (b) $(3, 5]$ excludes 3. <i>Trap</i> : the open bracket at 3; also the numerator zeros ± 1 are irrelevant. 3 each.
13	2	$-1 - 2 + 5$. <i>Trap</i> : sign of $(-1)^3$ and $-2(-1)^2$.
14	Yes; $c = 5$	2 continuity; 2 $f(2) = -8 < -5 < 7 = f(7)$; 2 solve $x^2 - 6x + 5 = (x - 1)(x - 5) = 0$, keep 5, reject 1.
15	Yes; $c = 3$	s is continuous on $[1, 4]$, $s(1) = 5$, $s(4) = -4$, and 1 is between. The graph passes through $(3, 1)$. 2 answer, 2 reason.
16(a)	F	IVT gives <i>at least</i> the values between $f(2)$ and $f(6)$; it does not rule out others. Counterexample: $f(x) = 4 + 10 \sin\left(\frac{\pi(x-2)}{4}\right)$, or any hump reaching above 10.
16(b)	F	“At least one,” not “exactly one.” Counterexample: a continuous zigzag from $(-1, -2)$ up to $(-\frac{1}{2}, 1)$, down to $(0, -1)$, up to $(1, 3)$ crosses 0 three times.

Points: $14 + 6 + 5 + 5 + 6 + 9 + 10 + 3 + 6 + 6 + 4 + 6 + 2 + 6 + 4 + 8 = 100$.

Interval notation in IVT items. Q14 and Q16(b) use a closed interval $[a, b]$ for c ; Q15 and Q16(a) use the open interval (a, b) . The mix is deliberate. In every item the target value differs from both endpoint values, so no endpoint can be a solution and the choice of bracket never changes an answer.